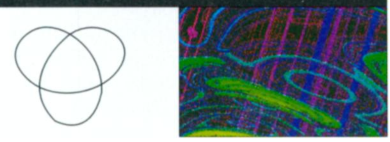
of its individual parts of this little-known prehistoric population. The detailed presentation of data in numerous tables and figures lends additional value to this volume, which should benefit other researchers investigating similar questions. These studies also make a significant contribution to the broader discipline of anthropology by illuminating fundamental oversimplifications in current characterizations of hunter-gatherer lifeways. Life at Stillwater Marsh was neither Hobbesian—nasty, brutish and short—nor utopian—the original affluent society—neither predominantly sedentary nor relentlessly mobile. Men and women made different contributions and suffered different pains for them. I wholeheartedly recommend this outstanding volume to both general and professional audiences.—*Mary Lucas Powell, Anthropology, University of Kentucky*

MATHEMATICS AND COMPUTER *sciences*



The World According To Wavelets. Barbara Burke Hubbard. A. K. Peters, Ltd., 1996. 264 pp. $34.

Wavelet theory is based on the idea of multi-scale analysis—a concept that is already familiar to many nonscientists through the popularization of fractals. Wavelets are sets of functions that can be transformed into each other by translation (shifting) and dilation (stretching or squeezing). In less than 10 years, wavelet analysis has had a major impact on such diverse fields as signal processing, where the applications include image compression, edge detection, and separation of noise from signal; numerical analysis, where it has led to more efficient algorithms; the study of turbulence; astronomy, where it has been used to identify new structures in clusters of galaxies; and medical imaging. When the FBI adopts a wavelet-based standard for digital coding of fingerprints, the time is ripe for a semipopular account of the subject.

In wavelet theory, the traditional Fourier-based approach of time-frequency analysis is replaced by one of time-scale analysis. The result is a tool, which has been described as a mathematical microscope, that uses closely spaced fine-scale functions to zoom in on areas of detail while concisely analyzing the rest with a few broad-scale functions. Much of the progress in wavelet theory is due to the development by Stéphane Mallat and Yves Meyer of a unifying framework known as multiresolution analysis. Interest then exploded when Ingrid Daubechies succeeded in constructing a set of wavelets in which the basic "mother" wavelet was smooth and so "compactly supported" that it was zero on all but a small interval, yet the different self-similar wavelets in the set were orthogonal, that is they canceled out in a way that greatly simplified many of the associated mathematical formulas. It was as if major advances in optics suddenly made high-powered microscopes widely available.

But wavelets are more than a framework for multiscale analysis; they also provide good time-frequency resolution of signals and have led to new methods of time-frequency analysis. Despite the conceptual simplicity of multiscale analysis, Hubbard follows the usual route of beginning her story with a review of the development of Fourier analysis, going back to Fourier's solution of the heat equation. Most scientists and engineers will find this superfluous, whereas nonscientists may be put off before reaching the more intuitive description of wavelet analysis in chapters two to four. Those who persist will find a lively and interesting story unfolded there.

Hubbard, who is not a mathematician, relies heavily on personal interviews, which sometimes seemed overly one-sided. For example, she states that the "best basis" fingerprint algorithm developed by Coifman and Wickerhauser "outperformed other methods," but, because it was being patented, "the FBI did not adopt it but instead custom-made a very similar technique." Why did she not check this with Tom Hopper at the FBI or Chris Brislawn at Los Alamos? Had she done so, she would have learned that the adaptive "best basis" algorithm took four to five times as long to achieve a quality *comparable* to the other algorithms tested and that the one adopted was developed *independently* using a fixed-partition biorthogonal wavelet decomposition with scalar quantization. (Both the Coifman/Wickerhauser algorithm and the FBI/Los Alamos algorithm differ from standard wavelet schemes by using nonoctave partitions. However, the methods used to determine the partitions are quite different.)

Her account of Daubechies's wavelet construction is even more disturbing. She quotes Yves Meyer as saying "Mallat launches brilliant ideas that keep two or three hundred people busy, then he goes on to something else. It was Ingrid Daubechies, with her tenacity, her capacity for work, who implemented it." Every mathematician to whom I showed that paragraph was astonished that anyone would reduce Daubechies's breakthrough to mere crank-turning (with hardly a mention of her many other contributions to the subject). Even though the iterative process Daubechies used was present in Mallat's multiresolution work, it was far from obvious that it could be used to construct smooth, compactly supported orthogonal wavelets. In fact, many experts did not even think such constructions were possible. Moreover, Mallat's work was itself based upon the Laplacian pyramid scheme developed earlier by Burt and Adelson.

Hubbard seems to have forgotten that when you look for the human side of a story, you will find mathematicians behaving like the humans that they are. She has told an engaging story, but one that very much reflects the views of the small circle of people she interviewed. Parts of wavelet analysis can be found in earlier work of many others, including the renormalization group of quantum field theory, the abstract mathematical work of Littlewood-Paley and Calderon-Zygmund, the stationary subdivision algorithms of approximation theory, the exact reconstruction filter banks of Smith and Barnwell, and other work of signal-processing engineers. As interest in wavelets grew, claims and counter-claims to priority were inevitable. Unraveling them would challenge a trained historian and, as Hubbard observes in her preface, correct attribution is further complicated by the difficulty of evaluating the impact of informal remarks and conversations. Even the seasoned journalist James Gleick slipped when, in his book *Chaos*, he gave the "Santa Cruz collective" full credit for the so-called "time delay method," although their original paper acknowledges the use of a suggestion of David Ruelle. Journalists should certainly strive for accuracy, but readers should be cautious about relying on them for either technical precision or historical attribution.

The nontechnical narrative is followed by a long supplement describing some of the underlying mathematics. In such discussions it may sometimes be necessary to use half-truths or oversimplifications to convey difficult concepts to nonspecialists. Presenting a good overview of a subject requires thorough knowledge and deep insight; it is much harder than giving a detailed explanation of a complex proof to specialists. My problem with the technical half of Hubbard's book is not the inevitable inaccuracies, but that she doesn't have the mathematical expertise and perspective needed to know when to gloss over subtleties, when to be precise and when to leave something out. Thus I admire her valor in attempting to present an introduction to quantum mechanics in 10 pages. But why didn't she simply say that quantum mechanics associates the physical quantities of position and momentum with operators that are related by Fourier transform so that the mathematical uncertainty principle applied here to time and frequency yields an analogous relationship between position and momentum in quantum theory? That would have sufficed.

The second half mixes undergraduate mathematics with expanded discussion of some of the more recent developments in wavelet theory, such as multiwavelets. I found the elementary sections generally well done and potentially useful; however, I am skeptical that those who would skip equations in a text would bother to read it. Moreover, words and equations are not the only alternatives. Both parts of this book make excellent use of illustrative diagrams to convey important concepts effectively. With perhaps even more reliance on figures and sidebars, much of what is in the second half could have been better integrated into the text; the rest could be omitted or put in an appendix.

Despite my disappointment with this book, it seems unfair to place too much of the blame on Hubbard's shoulders. Communicating with nonspecialists is an important responsibility that deserves far more attention from the mathematics community. Unless we are willing to meet this challenge ourselves, we can expect flawed efforts that attempt to fill the gap.—*Mary Beth Ruskai, Mathematics, University of Massachusetts, Lowell*

The Universe in a Handkerchief: Lewis Carroll's Mathematical Recreations, Games, Puzzles, and Word Plays. Martin Gardner. 158 pp. Copernicus, Springer-Verlag, 1996. $19.

Martin Gardner claims a spiritual kinship with Lewis Carroll. Through his books *Annotated Alice* (1960) and *More Annotated Alice* (1990), Gardner has introduced generations of modern readers to the mathematical allusions in Carroll's most famous literary works. In this new volume, he surveys the mathematical content of Carroll's other writings, including other fiction and verse, diaries and letters. Most of the second half of the book is devoted to the word games Carroll invented and published in articles and pamphlets, many of which are reprinted here for the first time. As such, this book should be received with enthusiasm by all those who share Gardner's fascination with the author of *Alice*.

It is good that this book has a subtitle, because the title is not very descriptive. In the middle of chapter one, "Fiction and Verse," there is a single paragraph referring to an example in "Sylvie and Bruno": "Mein Herr calls it Fortunatus's purse because, having neither inside nor outside, it can be said to contain the entire universe." In one of his columns in the old

"Birthday Quiz." Unfortunately, neither the world nor our editing is perfect, and we can say only that we made a mistake—one that cannot be blamed on Howard Topoff, the author of the quiz, because we wrote the title. Still, we commend Dr. Russell for careful reading, and as the winner of the unquiz he will receive a copy of *Exploring Evolutionary Biology: Readings from* American Scientist. Upon receiving this prize, we encourage Dr. Russell to turn to page *187*, where he can read about "The structure of natural selection."

Waves over Wavelets

To the Editors:

I read Mary Beth Ruskai's review (January–February) of *The World According to Wavelets* by Barbara Burke Hubbard with great interest. This book is based on interviews with several scientists who played some role in the wavelet saga. Being one of them, I feel responsible for two of the issues that Ruskai brought up in her review.

The first and most important issue concerns the role played by Stéphane Mallat and Ingrid Daubechies in the construction of smooth orthogonal wavelets with compact support. Daubechies created these wavelets in January–February 1987. Several months earlier—during the Fall of 1986—Mallat and I had a three-day discussion at the University of Chicago. Our conclusion was the following: Orthogonal wavelet bases, multiresolutional analysis and subband coding can be incorporated inside a unified formalism. (At that time, we were not aware that the filtering implemented in Mallat's algorithm was already known as subband coding in the electrical-engineering literature.)

Taking a historical perspective, Daubechies's construction belongs, therefore, to Mallat's program. Although this statement may be construed to mean that Daubechies merely treated a special example that arose from someone else's great vision, that was never my opinion. I always knew that a large gap existed between the formalism that I developed with Mallat and what Daubechies achieved. When one tries to apply this formalism, many problems occur immediately. The first one concerns the stability of the construction, and the second one is related to the desired regularity. These difficulties require a subtle and difficult analysis, which was achieved by Ingrid and could not be discovered by ordinary human beings.

I kept a beautiful seven-page letter that Daubechies wrote to me at the time she was working on her construction, explaining how she discovered the bases. Mallat's algorithm was not the starting point, but it did play a role as an ingredient.

The second issue concerns the FBI fingerprint contest and the role played by the best-basis algorithm in the proposal that was finally adopted for the standard. Hubbard reported a version of this story that she heard from me, and she should not be blamed if this is at variance with other reports.

Yves Meyer
Centre National de la Recherche Scientifique
Cachan, France